The Superintendent's Dilemma: Managing School District Capacity as Parents Vote with Their Feet

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Downsizing Urban Districts

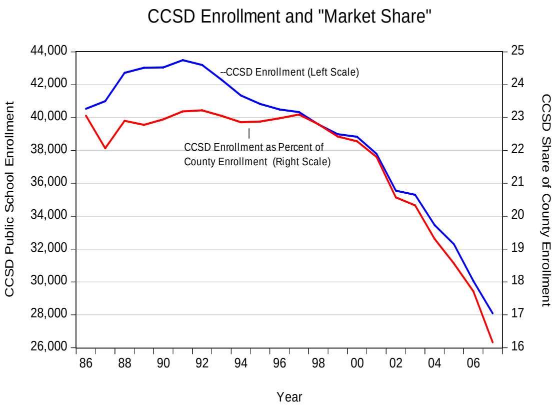
- Student retention has increasingly become an important issue for urban school districts in the U.S., especially in the East and Midwest.
- Public schools in many cities, including Buffalo, Chicago, Cincinnati, Cleveland, Detroit, Kansas City, Milwaukee, Pittsburgh, and Philadelphia, have lost tens of thousands of students over the last two decades.
- When policies aimed at retaining students are not successful, urban school districts are forced to downsize. Between 2001 and 2009, Chicago closed 44 schools.
- Detroit closed more than 100 schools over the last decade. Philadelphia recently announced closure of 23 schools.
- Kansas City, Mo, Milwaukee, Pittsburgh, and Washington closed between 20 and 30 schools each in recent years.

Motivation

- State funding for schools is typically allocated on a per-pupil basis.
- Faced with large excess capacity, districts are forced to close schools, trading off objectives such as student achievement and retention.
- In order to assess the efficacy of any policy to close schools, either retrospectively or prospectively, we need to consider how parents and students formulate their demand for schooling.
- There are important research questions that arise in the context of downsizing a school district that have not been studied in the previous literature. This paper attempts to fill this gap.

Contributions

- We examine a "central city" school district (CCSD) that was forced to drastically reduce capacity.
- We develop a sequential game in extensive form that captures the problems encountered in managing capacity.
- We estimate the parameters of our model exploiting our unique data set that follows students before and after the school closings.
- We evaluate the impact of the closing policy that was implemented in one district on school quality and student retention.
- We explore plausible alternative objective functions for the district to characterize the trade-offs faced by the administration.


CCSD Enrollment and "Market Share"
--CCSD Enrollment (Left Scale)
CCSD Enrollment as Percent of
County Enrollment (Right Scale)
CCSD Public School Enrollment
CCSD Share of County Enrollment
44,000
42,000
40,000
38,000
36,000
34,000
32,000
30,000
28,000
26,000
25
24
23
22
21
20
19
18
17
16
86
88
90
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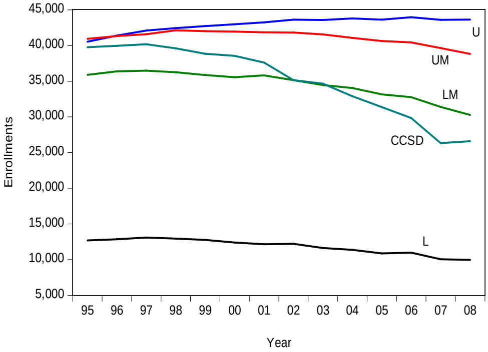
Year

Enrollment Trends in County School Districts
Suggest That Parents are Voting with Their Feet



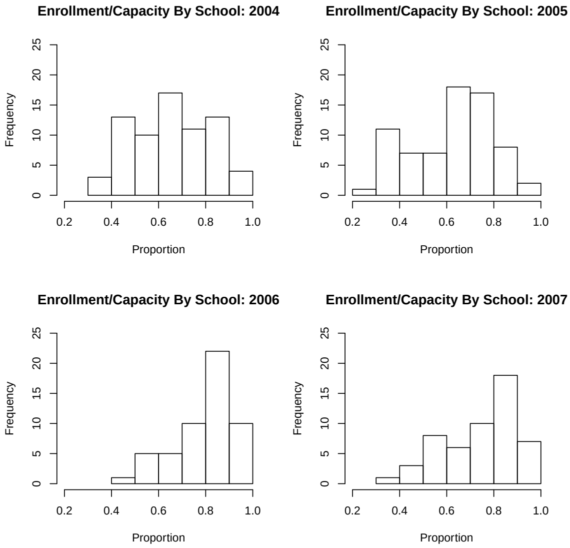
An Interpretation of Enrollment Patterns

- When countywide enrollment was rising, the district maintained its student share.
- When countywide enrollment began to decline in 1998, the district experienced two sources of enrollment decline:
    1. The district not only shared in the countywide decline in the student population, but
    2. The district experienced further decline as more affluent households exited the city and moved up the school district income hierarchy (voting with their feet)
- The combination of these two resulted in the district bearing 75 percent of the countywide decline in public school enrollment.

The "Right-sizing" Plan

- Countywide enrollment began to decline in 1998 largely to demographic factors.
- We find that 75 percent of the countywide decline in public school enrollment is suffered by the central city school district.
- In between the 2005-06 and 2006-07 school years, the district closed 22 out of 71 elementary and middle schools, reassigning 5227 students.
- This "right-sizing plan" used student-achievement data to dictate which schools to close.
- We find that the district had an overall capacity of 34,053 K-8 students in 2005 before the school closing.
- For grades K-5 (6-8), the capacity was reduced from 13,192 (20,861) to 9037 (15,550).

We see that the proportion of capacity utilized increases drastically post-closing.



Data

- Our sample consists of all K-8 students that are part of the CCSD database between 2004 and 2007.
- The data base includes some information for all school-ae children in the school district.
- We exclude all private school students that never attended a public school in our sample.
- We also eliminate 9-12 grade students from our sample since high schools were not affected by the right-sizing plan adopted in 2005.

Variables

- Student-level covariates: free or reduced lunch (FRL), race (White, Black, other), individual achievement, suspension days.
- School-level covariates: fraction FRL, fraction of each race, mean achievement, mean suspension days.
- Student-School level covariates: moving costs and travel costs (driving times)
- We do not have matched teacher - student data. Hence, we do not have teacher quality data at the class room level.
- We do observe the quality measure used by the district to close schools, but we do not have other teacher or principal quality measures at the school level.

Table of Summary Statistics

Students affected by School Closing

- There are 5227 students in the sample that were enrolled in the academic year 2005/06 in CCSD and attended schools that were subsequently closed.
- 4370 of those 5227 students were in K-7, and 422 of these 4370 left the district (9.7%).
- All of these students were assigned to attend one of of the public schools that remained open.
- However, parents and students exercised school choice. Hence, many students did not enroll at the assigned schools.
- We will use the assignment rules to construct predicted peer effects that can be used as instruments.

A Dicrete Choice Demand Model

- We develop a demand model in which households can choose to send the children to one of a variety of schools.
- We treat each public school as a differentiated product.
- We include four different outside options: charter schools, parochial schools, independent schools, and suburban schools.
- We assume that each public school as well as each of the four outside options differ by observed and potentially unobserved characteristics.
- School closings thus affect the number of public schools in the choice set as well as the characteristics of the school options.

Modeling the Choice Set

- Let J_t denote the set of potential public schools that are available at time t.
- A school configuration is given by a set of schools that are active. Let $J_t^o \subseteq J_t$ denote a potential school configuration.
- Similarly, let J_t^c denote the set of schools that are closed, andy by construction we have $J_t = J_t^o \cup J_t^c$.

Schools

- We treat each school as a differentiated product.
- Each school is characterized by a vector of (endogenous) peer effects, x_{jt}, and exogenous (unobserved) characteristics, ξ_{jt}.
- Endogenous characteristics are those that depend on the outcome of the student sorting process.
- Exogenous characteristics are, for example, the quality of the principal and the teachers.

Students and Parents

- Let z_{it} be the observed vector of characteristics of student i at time t.
- Let d_{ijt-1} be an indicator variable which is equal to one if student i attended school j in period $t-1$.
- Parents have private information that can be characterized by a vector of idiosyncratic choice specific shocks, denoted by ϵ_{jt}, and vector of preferences over endogenous characteristics, denoted by β_i.
- A student is completely characterized by a vector of characteristics $(z_{it}, d_{it-1}, \beta_i, \epsilon_{it})$.

Students

- We assume that the utility function of student i at time t is additively separable in the idiosyncratic preference shocks and can thus be written:

$$U_i(x_t, \xi_t, z_{it}, d_{it-1}, \beta_i, \epsilon_{it}) = \sum_{j \in J_t^o \cup O_t} d_{ijt} \left[u(x_{jt}, \xi_{jt}, z_{it}, d_{it-1}, \beta_i) + \epsilon_{ijt} \right]$$

- Given beliefs about endogenous qualities, the optimal strategy is then to choose the school that maximizes utility given the set of available school options.

A Parametrization of Preferences

Utility of individual i in school j in year t is:

$$U_{ijt} \quad = \quad \sum_{k=1}^{K} x_{jkt}\beta_{ikt} + \xi_{jt} + \epsilon_{ijt}$$

where the $x'_{jkt}s$ are observed school characteristics, the ξ_{jt} captures unobserved school characteristics, and a_{ijt} are covariates that vary by both student and school.

$$\beta_{ikt} \quad = \quad \alpha_{0k} \; + \; \sum_{l=1}^{L} \alpha_{1kl} \; z_{ilt} \; + \; \sigma_k \; v_{ik}$$

and the z_{ilt}'s are individual-level characteristics. The random coefficient errors satisfy: $v_{ik} \sim N(0,1)$.

Utility Characterization

Define the fixed effect of school j in year t as:

$$\delta_{jt} \;=\; \sum_{k=1}^{K} \alpha_{0k}\, x_{jkt} \;+\; \xi_{jt}$$

We can then write the school specific utility of individual i in year t as:

$$U_{ijt} \;=\; \delta_{jt} + \sum_{k=1}^{K}\sum_{l=1}^{L} \alpha_{1kl}\, x_{jkt} z_{ilt} \;+\; \sum_{k=1}^{K} \sigma_k\, x_{jkt} v_{ik} \;+\; \epsilon_{ijt}$$

Conditional Choice Probabilities

- We assume that the idiosyncratic shocks in the utility function follow a Type I extreme value distribution (McFadden, 1974).
- The conditional choice probabilities are given by:

$$
\begin{aligned}
Q_{ijt} &\equiv Pr\{d_{ijt} = 1 | v_i, X_t, Z_{it}, d_{it-1}\} \\
&= \frac{exp(\delta_{jt} + \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{1kl} x_{jkt} z_{ilt} + \sum_{k=1}^{K} \sigma_k x_{jkt} v_{ik})}{\sum_m exp(\delta_{mt} + \sum_{k=1}^{K} \sum_{l=1}^{L} \alpha_{1kl} x_{mkt} z_{ilt} + \sum_{k=1}^{K} \sigma_k x_{mkt} v_{ik})}
\end{aligned}
$$

Adapting the model for panel data

- We implicitely assume that student's current school choice only depends on the school they attended last year, not their whole history of school choices.
- We model choices for four time periods (T=4):

$$Pr\{d_{ij1}, d_{ik2}, d_{il3}, d_{im4} | v_i, X_t, Z_{it}, d_{in0}\} = Q_{ij1}\ Q_{ik2}\ Q_{il3}\ Q_{im4}$$

- Given that we do not observe U_i:

$$Pr\{d_{ij1}, d_{ik2}, d_{il3}, d_{im4} | X_t, Z_{it}, d_{in0}\} = \int Q_{ij1} Q_{ik2} Q_{il3} Q_{im4}\ dF(v_i)$$

- Further, we assume that each of these four random coefficients are time invariant, independently normally distributed, and approximate these distributions using quadrature methods (Skrainka and Judd 2011).

A Two Stage Estimator

- The conditional choice probabilities depend on the parameters α_1, σ, and the mean utilities $\delta_{1t}, .., \delta_{Jt}$.
- The likelihood function is given by

$$L = \Pi_{i=1}^{N} \int Q_{ij1} Q_{ik2} Q_{il3} Q_{im4} \, dF(v_i)$$

- The first stage of our algorithm yields an estimator of the school specific fixed effects denoted by $\hat{\delta}_{jt}$. Recall that:

$$\delta_{jt} = \alpha_0 \, x_{jt} + \xi_{jt}$$

- Following Berry (1994), we can estimate α_0 in the second stage using an IV estimator.
- We considered instruments derived from lagged peer effects and Hoxby & Weingarth (2006) style instruments that exploit exogenous assignment rules.

Some Comments

- We explicitly account for “habit formation” or moving costs in the model specification via conditioning on the previous year’s school choice.
- We also include distance to each school in order to capture the opportunity costs of commuting.
- We focus on four school characteristics: mean student achievement, fraction black, fraction FRL and mean suspensions.

First Stage Estimates

Sample	No Fixed Effects No Random Coefficients Full		Fixed Effects No Random Coefficients Full		Fixed Effects Random Coefficients Initial Conditions	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
School Achievement × Achievement	1.3433	0.0374	1.3715	0.039	2.0507	0.0847
School Achievement × Black	-0.1861	0.0647	-0.2908	0.073	-0.7057	0.1446
School Achievement × FRL	0.5255	0.0651	0.6083	0.072	0.2870	0.1402
School Black × Achievement	0.7657	0.0368	0.6855	0.0428	0.5725	0.0803
School Black × Black	0.7657	0.0368	3.0924	0.0850	5.914	0.1634
School Black × FRL	-0.6481	0.0660	-0.6243	0.0825	-0.3603	0.1381
School FRL × Achievement	0.4941	0.0391	0.3075	0.0486	0.5090	0.0958
School FRL × Black	-1.8534	0.0716	-0.8734	0.0839	-0.4846	0.1634
School FRL × FRL	2.4717	0.0777	3.9485	0.0860	5.755	0.1720
School FRL × Suspensions	0.1143	0.0064	0.1359	0.0095	0.1402	0.02618
School Suspensions × Achievement	0.0150	0.0065	0.0293	0.0101	0.0243	0.0240
School Suspensions × Black	-0.1282	0.0136	-0.0615	0.0203	-0.0353	0.0457
School Suspensions × FRL	0.0809	0.0144	0.0613	0.0204	0.0425	0.0477
School Suspensions × Suspensions	0.0154	0.0010	0.0320	0.0012	0.0643	0.0043

First Stage Estimates (cont)

	No Fixed Effects No Random Coefficients Full Sample		Fixed Effects No Random Coefficients Full Sample		Fixed Effects Random Coefficients Initial Conditions Sample	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Moving Costs	-4.4206	0.0296	-4.3259	0.0241	-4.177	0.0518
Moving Costs $\times$ Achievement	-0.2953	0.0186	-0.2766	0.0136	-0.2185	0.0316
Moving Costs $\times$ Black	-0.0736	0.0327	-0.1650	0.0250	0.1502	0.0524
Moving Costs $\times$ FRL	0.1992	0.0333	0.1882	0.0263	0.1319	0.0569
Moving Costs $\times$ Suspensions	0.0421	0.0039	0.0500	0.0027	0.0500	0.0086
Travel Times	-0.3716	0.0041	-0.4223	0.0033	-0.3747	0.0049
Travel Times $\times$ Achievement	-0.0017	0.0029	0.0003	0.0017	-0.0319	0.0030
Travel Times $\times$ Black	0.1118	0.0046	0.1092	0.0034	0.0652	0.0052
Travel Times $\times$ FRL	-0.1226	0.0047	-0.1217	0.0035	-0.0685	0.0055
School Achievement $\times$ RC					3.082	0.0997
School FRL $\times$ RC					3.941	0.1207
School Black $\times$ RC					3.396	0.0968
School Suspensions $\times$ RC					0.4447	0.047
Likelihood	-160,902		-136,777		-74,763	

Second Stage Estimates

	No Random Coefficients			Random Coefficients	
	OLS	Lagged Regressors 2SLS	Hoxby & Weingarth 2SLS	Lagged Regressors 2SLS	Hoxby & Weingarth 2SLS
School FRL	-3.478	-3.063	-3.891	3.294	-3.877
	(0.457)	(0.507)	(0.459)	(1.129)	(0.570)
School Black	-1.713	-1.707	-1.446	-5.233	-4.045
	(0.163)	(0.168)	(0.216)	(0.542)	(0.273)
School Achievement	-0.441	0.061	0.150	10.746	4.102
	(0.279)	(0.292)	(0.347)	(3.796)	(1.563)
School Suspensions	-0.171	-0.145	0.015	-0.118	-0.229
	(0.031)	(0.032)	(0.056)	(0.031)	(0.052)

Both specifications control for elementary school and middle school fixed effects.
Robust Standard Errors Included in Parentheses

Figure: Distribution of School Ability

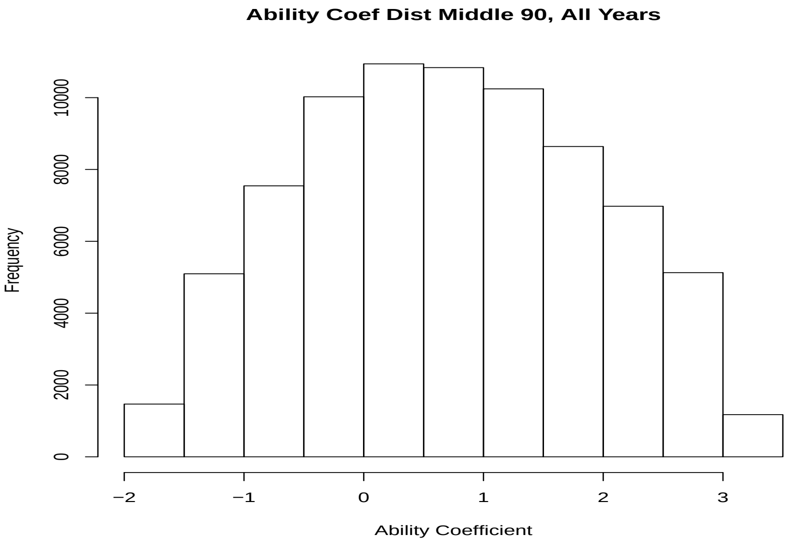
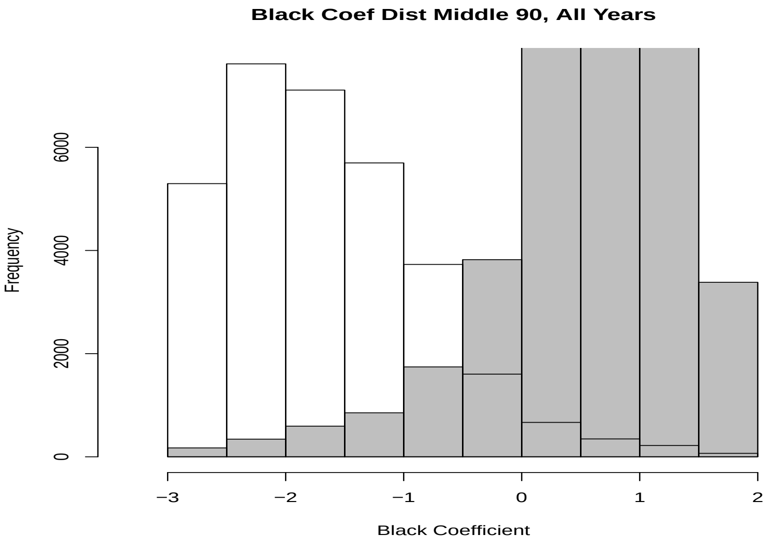
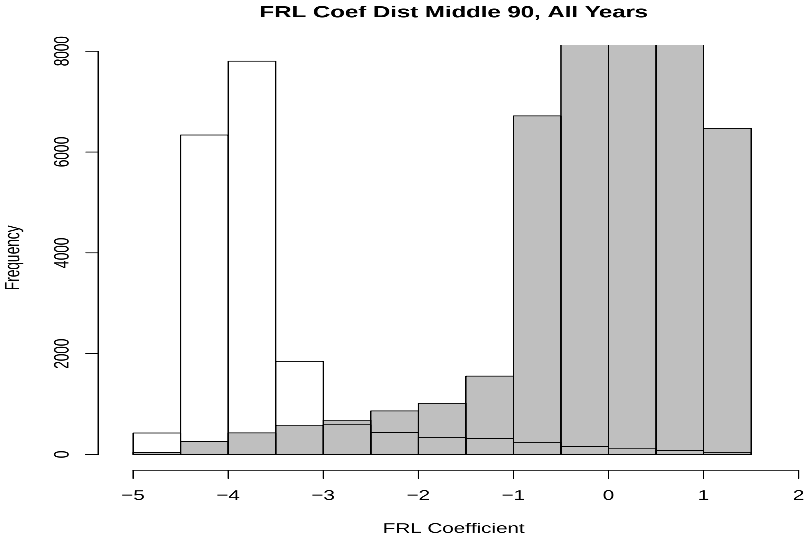


Figure: Distribution of School Black Coefficient: Evidence of sorting



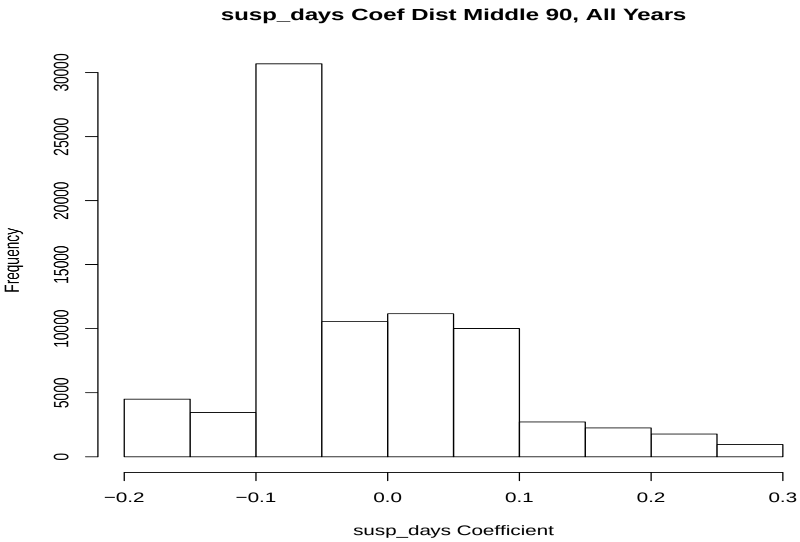
The shaded area corresponds to individuals who are Black, while the unshaded area corresponds to the individuals who are not Black.

Figure: Distribution of School FRL Coefficient: Evidence of sorting



The shaded area corresponds to individuals who receive free or reduced lunch, while the unshaded area corresponds to those individuals who do not.

Figure: Distribution of School Suspensions



Estimated School Quality and School Closings

	OLS	2SLS
Closed School	-1.693	0.150
	(0.344)	(0.178)
School FRL		-4.054
		(0.638)
School Black		-4.046
		(0.271)
School Achievement		3.746
		(1.682)
School Suspensions		-.238
		(0.052)

We use the panel-data random coefficients model for both specifications.
We use the Hoxby and Weingarth instruments for the IV specification.
Both specifications control for school year level fixed effects.
Both specifications control for elementary school and middle school fixed effects.
Robust Standard Errors Included in Parentheses

Distance and Choices

Variable	Prop. Attended School Within Quantile
Driving Times $\leq$ 25 % Quantile of Driving Times	0.694
Driving Times $\leq$ 50 % Quantile of Driving Times	0.795
Driving Times $\leq$ 75 % Quantile of Driving Times	0.981

Managing School Capacity

- We consider a sequential game in extensive form between a superintendent of the school district and a continuum of potential parents.
- In the first stage of the game, the superintendent determines the set of schools that are operational.
- In the second stage, parents enroll their children in one of the public schools or one of the outside options, taking into consideration that peer characteristics in schools are endogenous.
- Parents have idiosyncratic preferences for schools that are private information.
- The equilibrium concept that we use is Perfect Bayesian Equilibrium (PBE).

Equilibrium in the Second Stage

A set of strategies for each parent type implies an allocation (J_t^o, x_t^o, ξ_t^o) which is an equilibrium of the second stage of the game, if and only if the vector x_t^o satisfy the following requirement:

$$x_{jk}^o = \bar{z}_{jk} = \frac{\int z_k Pr\{d_{jt} = 1 | x_t^o, \xi_t^o, z, d_{t-1}\} \, f(z, d_{t-1}) \, dz \, dd_{t-1}}{\int Pr\{d_{jt} = 1 | x_t^o, \xi_t^o, z, d_{t-1}\} \, f(z, d_{t-1}) \, dz \, dd_{t-1}} \quad \forall k, j$$

The District's Objective Function

- Managing school capacity often attempts to identify and close "under-performing" schools.
- To formalize these ideas, suppose the district observes a quality measure, denoted by q_j, for each school.
- We assume that the quality measure depends on the school characteristics as well as on some exogenous variables w_j that may not affect household sorting, i.e. $q_j = q_j(\xi_j, x_j, w_j)$.
- A reasonable objective of the school district is then to maximize the weighted average of school quality:

$$Q \;=\; \sum_{j \in J^o} s_j \; q_j(\xi_j, x_j, w_j)$$

Feasebility and Capacity Constraints

- The district then maximizes Q over the set of feasible school configurations that result in an equilibrium of the second stage of the game.
- In addition, total excess capacity in the district does not exceed a given threshold:

$$\sum_{j \in J^o} (n_j - s_j) \leq c.$$

A Perfect Bayesian Equilibrium

We say that a set of strategies that imply an allocation, $(J_t^{o*}, x_t^{o*}, \xi_t^o)$, is a PBE of our game in extensive form, if and only if,

a) for each possible school configuration J_t^o there exists a Bayesian Nash equilibrium;

b)$(J_t^{o*}, x_t^{o*}, \xi_t^o)$, is a Nash equilibrium for school configuration J_t^{o*}; and

c) Given all second stage equilibria given in (a), there does not exist another equilibrium in the second stage and yields higher welfare for the superintendent and satisfies the capacity constraint.

Existence and Uniqueness

- There exists an equilibrium of the second stage of the game. (Prop 1)
- If there are multiple equilibria in second stage subgame for a given set of of schools J_t^o, there generically exists a unique equilibrium that dominates all other PBE according to the welfare criterium of the superintendent. (Prop 2)
- There exists, at least, one PBE for the game in extensive form.

Solution Algorithm

- The algorithm consists of two loops.
- For a given set of schools that remain open, i.e. a given set J^o, the inner loop determines a feasible school configuration.
- The outer loop of the algorithm then searches over all feasible combinations of school closings and finds the one that minimizes the objective function.
- This algorithm is computationally practical if the number of schools to be closed is not too large.
- In the application, we consider the case of closing three middle schools and reducing capacity by five percent.
- We have recast the problem of computing an equilibrium as finding the solution to a non-linear integer programming problem.

Alternative Objective Functions

- Inequality: the weighted squared deviation between school j and district characteristics is then given by the following index

$$I = \sum_{j \in J^o} \sum_{l=1}^{L} \omega_l \left(\bar{z}_{jl} - \bar{z}_l^d\right)^2$$

  where ω_l the weight assigned to school characteristic l.
- Retention: number of students staying in the district.

$$R \;=\; \sum_{j \in J^o} \sum_{i} \sum_{s} f_{is}\; p_{isj}$$

- Disruption: number of students in closed schools.

$$D \;=\; \sum_{j \in J^c} \sum_{i} \sum_{s} f_{is}\; p_{isj}^{baseline}$$

Binding Capacity Constraints and Rationing

- Each pubic school has binding capacity constraint that is equal to n_j.
- Equilibrium in the second stage of the game must satisfy the school capacity constraints:

$$\int Pr\{d_{jt} = 1 | x_t^o, J_t^o, z, d_{t-1}\}\, f(z, d_{t-1})\, dz\, dd_{t-1} \leq \; n_j$$

- We assume that public school j has shadow admission price denoted by p_j. It is straight-forward to extend our utility function and obtain:

$$\sum_{j \in J_t \cup O_t} d_{ijt} \left[u(x_{jt}, p_{jt}, \xi_{jt}, z_{it}, d_{it-1}, \beta_i) + \epsilon_{ijt} \right]$$

The Effects of School Closing

- We can classify schools based on the estimated fixed effects. Note that these fixed effects capture parental perceptions of school quality. They are not directly based on achievement measures, for example.
- In the second stage, we condition on observed peer characteristics, in order to see if these facts hold up conditionally.
- Closed schools have lower quality (fixed effects) than schools that remained open.
- Once we condition on observed differences in peer quality, there is no evidence that closed schools had systematically worse unobserved characteristics than other public schools.

Optimal School Closing Analysis

		1	2	3	4	5
Pre-sorting School Market Outcomes						
		Baseline	Quality	Diversity	Retention	Dislocation
a	Enrollment: Closed Schools	0	559	330	477	63
b	Mean FRL	0.70	0.68	0.70	0.73	0.69
c	Mean Black	0.60	0.58	0.60	0.64	0.58
d	Mean Achievement	-0.80	-0.82	-0.48	-0.70	-0.24
e	Mean Suspensions	5.01	4.86	2.87	5.33	4.61
f	Enrollment: Outside Options	1444	1605	1511	1245	1333

All means and standard deviations are for the remaining open public schools.
We use the baseline probabilities for the pre-sorting panel.
This table is for students in grades 6-8; $N = 8,245$

Summary of Findings

- The retrospective analysis shows that parents primarily care about peer characteristics and proximity to school.
- Though closed schools were of overall lower quality, these differences in quality relative to other public schools disappeared once we condition on observed school characteristics.
- From our prospective example of an optimal school closing formulation, we find that the district can reduce excess capacity without lowering school quality.
- However, the retention of students is a more difficult objective.

Table: Summary Statistics of the Sample K-8

	2004	2005	2006	2007
Sample Size	24660	24876	24489	23735
Sample Size: Public + Private	22158	21189	20333	19045
Sample Size: Public	21239	20180	19189	17956
Free or Reduced Lunch	17438	17840	17446	16866
Race: Black	14256	14219	13866	13312
Race: White	8498	8545	8405	8164
Race: Other	1906	2112	2218	2259
Median Household Income	28108	28318	28425	28659
Median Housing Value	56663	57178	57615	57929
Moving choices indicator	10938	9920	12255	8796
Driving times to school attended	3.086	3.130	3.273	3.362
Individual achievement measure	-0.042	-0.033	-0.031	-0.045
Number of Suspension days	1.234	1.168	1.367	1.031