# Notes on Dynamic Owner-Occupied Housing Demand

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# Owner-Occupied Housing Demand

- Here we sketch the dynamic housing demand model that was developed by Diaz and Luengo-Prato (2010) and estimated by Bajari, Chan, Krueger and Miller (2013). A similar model is used by Hurst, Keys, Seru, and Vavra (2016).
- Let's ignore renters and assume that everybody is an owner.
- We model a typical household's consumption and housing choice as a partial equilibrium, dynamic decision problem with a finite lifetime horizon.
- Households live for $T$ periods, and in each period t they choose consumption expenditures on nondurables, $c_t$, and the amount of one-period risk-free financial assets (bonds) to bring to the next period, $b_{t+1}$.

# Preferences

- Let $h_t$ denote the size of the household's owned real housing stock brought into the period, so that $h_{t+1}$ is the amount of housing chosen for today.
- Households value nondurable consumption, $c_t$, and housing, $h_{t+1}$, according to the period utility function

$$u(c_t, h_{t+1}) = \ln[(\theta \, c_t^{\rho} + (1-\theta)(e^{\kappa_t} \, h_{t+1})^{\rho})^{1/\rho}] \quad (1)$$

  where $\kappa_t$ is an iid preference shock.
- We need the preferences shock to generate changes in housing consumption when income does not change.

# Lifetime Utility

- Expected lifetime utility is then given by

$$E_0 \left[ \sum_{t=1}^{T} \beta^{t-1} u(c_t, h_{t+1}) + \gamma \beta^T (b_{T+1} + p_{T+1} h_{T+1}) \right] \quad (2)$$

  where $\beta$ is the standard time discount factor, $T$ determines the end of working life, and $\gamma$ measures the degree of altruism to leave bequests.
- Expectations are taken with respect to the stochastic processes driving labor income, housing preference shocks, and house prices, which we specify later.

# Law of Motion for Prices and Income

- Let $p_t$ denote the relative price of one unit of housing in terms of the numeraire nondurable consumption good.
- Housing prices follow first order stochastic Markov process.
- At time 0, agents are endowed with initial asset holdings $(b_0, h_0)$ and one unit of time per period, which they supply inelastically to the labor market to earn labor income $y_t$.
- The labor income process is composed of two components, a deterministic mean life-cycle profile $\epsilon_t$ (which incorporates income growth over the life cycle) and a stochastic component $\eta_t$ that follows a first order Markov process.
- Thus labor income is given by $y_t = \epsilon_t \eta_t$.

# Adjustment Costs and Taxes

- We model three main frictions in the housing market explicitly.
- First, the stock of housing is subject to non-convex adjustment costs. Specifically, in order to purchase a home of size $h_{t+1}$ the household has to spend $p_t h_{t+1}$ plus adjustment costs given

$$p_t \, \Phi(h_{t+1}, h_t) \;\; = \;\; \phi \, 1\{h_{t+1} \neq h_t\} \, p_t h_{t+1} \qquad (3)$$

- Second, the household has to pay property taxes. Assuming a time invariant property tax rate $\tau$, the tax payments in a market value tax system are given by $\tau \; p_t \; h_{t+1}$.

## Downpayment Requirement

- A third key friction in our model is the requirement for households to acquire and maintain some minimal positive equity share in the house.
- We assume that the joint choice of financial assets and housing positions satisfies the following collateral constraint:

$$b_{t+1} \geq -(1-\xi)p_t h_{t+1}. \tag{4}$$

  Here $\xi$ is the fraction of the purchase price of the house that has to be paid down at purchase, that is, $(1-\xi)$ is the fraction of the purchase price that can be financed via a mortgage.
- In most of our experiments we shall assume that households are able to finance at most 80% of their housing purchases through mortgages.
- Also note that as long as $\xi < 1$, households can only borrow against their housing collateral; uncollateralized debt is therefore ruled out by assumption in our model.

# Financial Assets and Budget Constraint

- In addition to housing, households can use financial assets to accumulate wealth. These assets yield a real interest rate $r_t$.
- If households borrow (subject to the collateral constraints), they face a real mortgage interest rate $r_m > r$.
- Defining $r(b) = r$ if $b \geq 0$ and $r(b) = r_m$ if $b < 0$, the budget constraint can be written as

$$\begin{aligned} & c_t + b_{t+1} + (1+\tau)\, p_t h_{t+1} + \phi\, 1\{h_{t+1} \neq h_t\}\, p_t h_{t+1} \\ = \;& y_t + (1 + r(b_t))b_t + p_t h_t. \end{aligned} \tag{5}$$

# Non-Negativity Constraints

- Finally, consumption and housing choices are constrained to be nonnegative:

$$c_t, \quad h_{t+1} \geq 0. \tag{6}$$

- Households maximize inter-temporal utility (2) subject to the constraints (4), (5), and (6).

# Recursive Representation

Following BCKM, the model in recursive formulation can be written as:

$$V(\kappa, \eta, b, h, p, t) \quad = \quad \max_{c, b', h'} \left[ u(c, \kappa, h') + \beta E V(\kappa', \eta', b', h', p', t+1) \right]$$

s.t.

$$\begin{aligned} c, \quad h' \quad & \geq \quad 0 \\ b' \quad & \geq \quad -(1-\xi) p h' \\ c + b' + (1+\tau) p h' + \phi \, 1\{h' \neq h\} \; p h' \quad & = \quad \eta \epsilon_t + (1 + r(b)) \; b + p h \end{aligned}$$

# Voluntary Equity

- As pointed out by DL and BCKM, the constraint set for $(b', h')$ is not rectangular, i.e. the constraint on $h'$ depends on $b'$, which is itself a choice variable.
- This problem can be overcome by defining a new variable called voluntary equity, $q'$, as:

$$q' \quad = \quad b' + (1 - \xi)\, p\, h' \tag{7}$$

- Voluntary equity is the wealth held by the households in excess of the required downpayment requirement at the beginning of the period.
- This definition implies that

$$q \quad = \quad b + (1 - \xi)\, p_{-1}\, h \tag{8}$$

where $p_{-1}$ is the price of housing in the previous period, which now becomes an additional state variable.

# New Recursive Representation

This transformation of variables gives us the following problem:

$$\begin{aligned} & V(\kappa, \eta, q, h, p_{-1}, p, t) \\ = & \max_{c, q', h'} \left[ u(c, \kappa, h') + \beta E \; V(\kappa', \eta', q', h', p, p', t+1) \right] \end{aligned} \tag{9}$$

With this reformulation, the model has two control (wealth) variables $h'$ and $q'$ that are constrained to be non-negative.

$$\begin{aligned} c & \geq 0 \\ h' & \geq 0 \\ q' & \geq 0 \\ & c + q' + \xi \; ph' + \phi \; 1\{h' \neq h\} \; ph' + \tau \; ph' \\ & = \eta \epsilon_t + (1 + r(\cdot)) \; q + [p - (1 - r(\cdot))(1 - \xi) p_{-1}] h \end{aligned} \tag{10}$$

where

$$r(q, h, p_{-1}) = \begin{cases} r & \mathit{if} \quad q - (1 - \xi) p_{-1} h \geq 0 \\ r_m & \mathit{if} \quad q - (1 - \xi) p_{-1} h < 0 \end{cases}$$

# Comments on Solving the Model

- Since we use an adjustment cost that is non-convex, the household decision problem is not a convex programming problem, and numerical approaches that require differentiability of the value function cannot be applied.
- Therefore we use discrete state space dynamic programming techniques to solve the problem. In particular, we discretize the state space for $(q, h)$ into a finite (but not evenly spaced) rectangular grid (the income and house price process is already a finite state Markov chain by assumption) and maximize the objective function by searching for each $(q, h)$ over the finite grid of admissible choices.
- The consumption choice is implied by the budget constraint.

# Comments (cont)

- Given a terminal value function (given by the bequest function), we can iterate backward in age of the household $t$ to solve for the age-dependent optimal policy and value functions.
- Once we have computed these, simulated life-cycle patterns of consumption, housing, and financial wealth can be generated for any sequence of house price and income shock realizations.
- Computation of a single policy function took week on an advanced workstation.

# References


- Bajari, P., Chan, P., Krueger, D. and D. Miller (2013). A dynamic Model of Housing Demand: Estimation and Policy Implications. International Economic Review, 54 (2). 409 - 442.
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